Azonosító jel:															

ÉRETTSÉGI VIZSGA • 2017. május 9.

MATEMATIKA ANGOL NYELVEN

EMELT SZINTŰ ÍRÁSBELI VIZSGA

2017. május 9. 8:00

Időtartam: 240 perc

Pótlapok száma	
Tisztázati	
Piszkozati	

EMBERI ERŐFORRÁSOK MINISZTÉRIUMA

Instructions to candidates

1. The time allowed for this examination paper is 240 minutes. When that time is up, you will have to stop working.

2. You may solve the problems in any order.

3. In part II, you are only required to solve four of the five problems. **When you have finished the examination, enter the number of the problem not selected in the square below**. *If it is not clear* for the examiner which problem you do not want to be assessed, the last problem in this examination paper will not be assessed.

4. On solving the problems, you may use a calculator that cannot store and display textual information. You may also use any edition of the four-digit data tables. The use of any other electronic device or printed or written material is forbidden!

5. **Always write down the reasoning used to obtain the answers. A major part of the score will be awarded for this.**

6. **Make sure that calculations of intermediate results are also possible to follow.**

7. **The use of calculators** in the reasoning behind a particular solution **may be accepted without further mathematical explanation in case of the following operations**: addition, subtraction, multiplication, division, calculating powers and roots, $n!$, $\binom{n}{k}$, replacing the tables found in the 4-digit Data Booklet (sin, cos, tan, log, and their inverse functions), approximate values of the numbers π and e, finding the solutions of the standard quadratic equation. No further explanation is needed when the calculator is used to find the mean and the standard deviation, as long as the text of the question does not explicitly require the candidate to show detailed work. **In any other cases, results obtained through the use of a calculator are considered as unexplained and points for such results will not be awarded.**

8. On solving the problems, theorems studied and given a name in class (e.g. the Pythagorean Theorem or the height theorem) do not need to be stated precisely. It is enough to refer to them by name, but their applicability needs to be briefly explained. Reference to other theorems will be fully accepted only if the theorem and all its conditions are stated correctly (proof is not required) and the applicability of the theorem to the given problem is explained.

9. Always state the final result (the answer to the question of the problem) in words, too!

10. Write in pen. Diagrams may be drawn in pencil. The examiner is instructed not to mark anything written in pencil, other than diagrams. If you cancel any solution or part of a solution by crossing it over, it will not be assessed.

11. Only one solution to each problem will be assessed. In case of more than one attempt to solve a problem, **indicate clearly** which attempt you wish to be marked.

12. Please, **do not write in the grey rectangles**.

I.

1. Solve the following inequalities in the set of real numbers.

a) $\log x < 2$

b) $4x < 5 - x^2$

c) $0.5^{|x-3|} < 0.25$

a)	3 points	
b)	4 points	
c)	5 points	
T.:	12 points	

2. Noémi’s first university exam consists of three parts: a project work, a written test, and an oral presentation. The results of all three parts are given in percentage form.
The final score on the exam is calculated as the weighted average of the three percentage results. The project work weighs 2 units, the test weighs 5 units, and the oral presentation weighs 3 units.
Noémi got 73% on her project work and 64% on her written test.

a) What percentage should she receive on the oral presentation if she needs a combined minimum of 70% on the whole exam?

Further analysis of the test results of first-year students revealed that the average result of the 75 girls taking the exam was 70%, while the average result of the boys taking it was 62%. Also, the average result of the 40 students staying at the university hall of residence was 71%, while the average of those living elsewhere was 65%.

b) How many first-year students took this exam altogether?

a)	4 points	
b)	7 points	
T.:	11 point	

3. The table shows the masses of 8 friends.

Name	Albert	Bori	Csaba	Dénes	Elek	Frigyes	Gabi	Helga
Mass (kg)	82	74	90	88	85	85	63	71

a) Give the median, mean, and the standard deviation of the 8 data points.

The 8 friends would like to take the elevator to the top floor of a building. The elevator is rather small, a note says: *"3 people or 230 kg max."* (i.e. no more than 3 people may ride the elevator at the same time, moreover, the total mass of those riding it may not exceed 230 kg).

b) Prove that the 8 friends can ride to the top floor in no more than three rounds (while not violating the regulations either).

After an overhaul the capacity of the elevator has been increased to 300 kg, but the restriction about the number of people still applies (no more than 3 people at the same time).

c) In how many different ways can the 8 friends get to the top floor, given that at least two of them ride the elevator together at any time and they keep the revised regulations, too? (Two possibilities are considered different if either there is at least one group in them that consists of different people, or the same groups travel to the top floor but in different order.)

a)	4 points	
b)	3 points	
c)	7 points	
T.:	14 points	

4. **a)** Calculate the area between the graphs of the parabola $y = -x^2 + x + 6$ and the line $x - y + 2 = 0$.

The parabola $y = -x^2 + x + 6$ intersects the x-axis in points A and B.

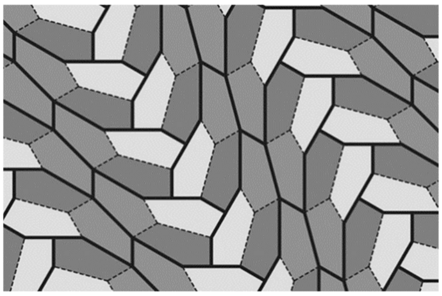
b) Calculate the gradient (slope) of the tangent line drawn to the parabola at point B, given that the first coordinate of point B is positive.

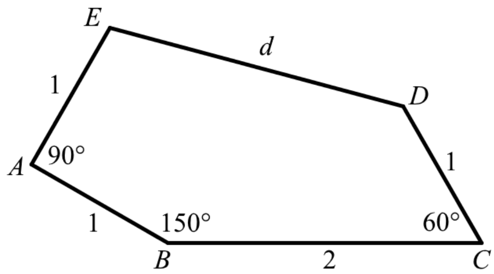
a)	8 points	
b)	6 points	
T.:	14 points	

II.

You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 2.

5. An interesting piece of internet news in 2015 was the discovery of a new way of filling the flat plane without gaps (tessellation) with congruent pentagons. (The first diagram shows a section of the tessellation, while the second provides further details about the pentagons: $EA = AB = CD = 1$, $BC = 2$, $EAB\angle = 90º$, $ABC\angle = 150º$, $BCD\angle = 60º$.)





a) Prove that the angle between the two diagonals drawn from vertex *B* of the pentagon is 75°.

b) Prove that $\cos 75° = \frac{\sqrt{6}-\sqrt{2}}{4}$ (you may also use the trigonometric addition formulas here).

c) Prove that the exact length of side *DE* of the pentagon is $\sqrt{2+\sqrt{3}}$.

d) Prove that $\sqrt{2+\sqrt{3}} = \frac{\sqrt{6}+\sqrt{2}}{2}$.

a)	5 points	
b)	3 points	
c)	5 points	
d)	3 points	
T.:	16 points	

You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 2.

6. **a)** The logical value of the statements *A* and *C* is true, statement *B* is false.
Determine the logical value of the following statements.
(**In this part** no further explanation is required.)

(1) $\neg A \vee \neg B$
(2) $(A \wedge B) \vee \neg C$
(3) $B \to \neg A$
(4) $\neg A \leftrightarrow B$
(5) $A \to (B \wedge C)$

Set *H* is the set of all 10-point, simple graphs. The following statement refers to the elements of *H*: *If a (10-point, simple) graph has at least 8 edges, then it does not contain any circuits.*

b) Decide, if the above statement is true or false. Explain your answer.

c) Write down the converse of the above statement for elements of set *H* and decide if the converse is true or false. Explain your answer.

Select three different edges randomly from among the edges of a 10-point complete graph. (A complete graph is a simple graph with an edge between any two points.)

d) Calculate the probability that the three selected edges form a circuit.

a)	3 points	
b)	3 points	
c)	4 points	
d)	6 points	
T.:	16 points	

You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 2.

7. **a)** How many different acute triangles are there whose angles, measured in degrees, are all different integers and form three consecutive terms of an arithmetic sequence? (Two triangles are considered different as long as they are not similar to each other.)

b) Prove there is no regular n-gon (n-sided polygon) whose interior angles are n degrees each.

c) The measure of the interior angles of a regular polygon, in degrees, are all integers. How many different values of n are possible?

a)	4 points	
b)	4 points	
c)	8 points	
T.:	16 points	

You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 2.

8. During an epidemic 0.2% of the population of a big city got infected by a virus. At one point in time 80 people living in the city travelled on the same bus.

a) Calculate the probability that at least one person out of the 80 travelling on this bus is infected. Round your answer to two decimal places.

Models, describing the spread of the epidemic in the city, forecast that each day the number of people infected will increase to 105% of the number of people who were infected the day before.

b) How many days would it take until the percentage of the infected population rises from 0.2% to 1%, assuming the epidemic spreads as predicted by the models?

A test, sold in pharmacies, promises users to quickly reveal whether they are infected or not. The description says: *"The test positively indicates infection at 99% probability for users truly infected. The test is also known to falsely indicate infection in case of users not infected. However, the probability of such false-positive result is only 4%."*

c) It is known, that 0.2% of the total population of the city is truly infected. A randomly selected citizen living in this city is being subject to the above test and the test indicates infection. Show that the probability of the person being truly infected in this case is less than 0.05 (and the test is therefore unsuited to reliably indicate infection).

a)	4 points	
b)	5 points	
c)	7 points	
T.:	16 points	

You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 2.

9. A total 350 tons of consumer goods are to be transported by railroad in more than one rounds. The fees of one of the transportation companies consist of two items: a transportation fee, which is proportional to the square of the total mass of the goods carried, must be paid as well as a fixed basic fee. Transporting a total t tons of goods through this company would cost $\frac{t^2}{10}+205$ Euros.

a) The 350 tons of goods are to be transported in two rounds (at two different occasions). Prove that the cost of transportation is lowest if the goods are carried in two parts of equal mass.

To lower the costs, the 350 tons of goods are divided into n parts of equal mass ($n \in \mathbf{N}^+$). One part at a time is going to be carried by rail.

b) Prove that the total cost by the above company for the n separate transports will be $\frac{12\,250}{n}+205n$ Euros!

Besides transportation fees, dividing the 350 tons of goods into n parts of equal mass would cost a further $(n-1)\cdot 400$ Euros ($n \in \mathbf{N}^+$).

c) How many parts of equal mass should the 350 tons of goods be divided into for a minimal total cost?

a)	4 points	
b)	3 points	
c)	9 points	
T.:	16 points	

	Number of problem	score			
		maximum	awarded	maximum	awarded
Part I	1.	12		**51**	
	2.	11			
	3.	14			
	4.	14			
Part II		16		**64**	
		16			
		16			
		16			
		← problem not selected			
Total score on written examination				**115**	

date

examiner

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	pontszáma **egész számra** kerekítve	
	elért	programba beírt
I. rész		
II. rész		

dátum

dátum

javító tanár

jegyző